II Semester M.Sc. Degree Examination, July 2017
(RNS – Repeaters) (2011-12 and Onwards)
MATHEMATICS
Paper – M-204 : Partial Differential Equations

Time : 3 Hours | Max. Marks : 80

***Instruction** : Answer **any five full** questions.*

1. a) Eliminate c from $x^2 + y^2 + (z - c)^2 = c^2$ to form the partial differential equation.

   b) Give an example for first order linear, semilinear, quasilinear and second order linear partial differential equations.

   c) Solve $u(x + y)u_x + u(x - y)u_y = x^2 + y^2$ with $u = 0$ on $y = 2x$. **(4+4+8)**

2. a) Obtain solution of $(p^2 + q^2)\, x = pz$ containing the curve $x = 0$, $z^2 = 4y$.

   b) Find the solution of i) $u_t + xu_x = -tu$, $u(x, 0) = \sin x$ and ii) $u_x u_y = u$ with $u(0, y) = y^2$ by using the method of characteristics. **(6+10)**

3. a) Classify the following equations as being hyperbolic or parabolic or elliptic

   i) $(n-1)^2 \dfrac{\partial^2 z}{\partial x^2} - y^{2n} \dfrac{\partial^2 z}{\partial y^2} = ny^{2n-1} \dfrac{\partial z}{\partial y}$,

   ii) $u_{xx} + 2u_{xy} + u_{yy} = 0$ and

   iii) $u_{xx} + xu_{yy} = 0$, $x \neq 0$.

   b) Reduce the equation $u_{xx} - u_{yy} = 0$ to its canonical form. **(9+7)**

4. Explain Monge's method of solving $F(x, y, z, p, q, r, s, t) = 0$ and using the method obtain the solution of the equation $r + 4s + t + (rt - s^2) = 2$. **16**

5. a) Obtain D'Alemberts' wave solution of $\dfrac{\partial^2 u}{\partial t^2} = 16 \dfrac{\partial^2 u}{\partial x^2}; -\infty < x,\ t < \infty$ subject to

$$\left.\begin{aligned} u(x,0) &= f(x) \\ \frac{\partial u}{\partial t}(x,0) &= g(x), \end{aligned}\right\} -\infty < x < \infty.$$

   b) Show that a variable separable solution of wave equation in spherical coordinates leads to a Legendre differential equation. **(6+10)**

P.T.O.

6. a) State and prove Dirichlet problem in a circular region.

   b) Obtain solution of Neumann problem in a Half-plane

   $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0;\ -\infty < x < \infty,\ 0 < y < \infty$$

   subject to

   $$\frac{\partial u}{\partial y}(x, 0) = f(x)$$

   $$\frac{\partial u}{\partial y} \to 0 \text{ as } y \to \infty$$

   (10+6)

7. a) Solve by appropriate Fourier transform the IBVP

   $$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2};\ 0 \le x \le \infty,\ t \ge 0$$

   subject to

   $$u(x,\ 0) = f(x) = \begin{cases} x; & 0 < x < 2 \\ 0; & \text{elsewhere} \end{cases}$$

   $$u(0,\ t) = 0;\ t \ge 0$$

   b) Solve by Fourier decomposition the following IBVP :

   $$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2};\ 0 \le x \le L, L \ge 0$$

   subject to

   $$u(x,\ 0) = f(x) = \begin{cases} \dfrac{x}{L}; & 0 \le x \le \dfrac{L}{2} \\ 1 - \dfrac{x}{L}; & \dfrac{L}{2} \le x \le L \end{cases}$$

   $$\left.\begin{matrix} u(0, t) = 0 \\ u(L, t) = 0 \end{matrix}\right\};\ t \ge 0$$

   (8+8)

8. a) Explain similarity transformation through an appropriate example.

   b) Using Green's function solve $u_t - u_{xx} = g(x)\ \delta(t);\ -\infty < x < \infty,\ t > 0$ subject to $u(x,\ 0) = 0;\ -\infty < x < \infty$.

   c) Write down a weak formulation of $u_{xx} + u_{yy} + u = 0$. **(6+5+5)**